Int. J. Heat Mass Transfer. Vol. 18, pp. 289–299. Pergamon Press 1975. Printed in Great Britain

A MODEL OF HEAT TRANSFER IN GAS FLUIDIZED BEDS

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(*Received* 12 *March* 1974 *and in revised form* 11 *June* 1974)

Abstract—The mechanism of heat transfer at surfaces in fluidized beds is discussed and a variant of the penetration theory is developed in which the packet theory of heat transfer is modified to allow for, and describe quantitatively, property variations in the packet in the region of the surface.

Property variations are described in terms of the voidage variations in the vicinity of a constraining surface which are modelled from simple geometrical considerations.

The model of the heat-transfer process derived here shows good agreement with experimental data without recourse to any of the semi-empirical approximations that are features of the previous models.

NOMENCLATURE

A_B,	bed cross-sectional area;
$A(x)$,	cross-sectional area of solid particles at point x;
b_{eq},	equivalent dimensionless conductivity;
$b(Z)$,	function defined by equation (11);
c_p,	specific heat;
$c_p(x)$,	specific heat of emulsion phase at point x;
C,	constant of proportionality;
d_p,	particle diameter;
f_s,	surface heat flux;
$fn(x)$,	function of x;
Fo,	$\kappa_B t/d_p^2$, instantaneous Fourier number;
Fo_m,	$\kappa_B \tau/d_p^2$, time-mean Fourier number;
$g(Z)$,	function defined by equation (10);
h,	heat-transfer coefficient;
$H(Z)$,	function defined by equation (12);
k,	thermal conductivity;
$k(x)$,	thermal conductivity of emulsion phase at point x;
L_d,	heat penetration depth;
$L(Z)$,	function defined by equation (13);
Nu,	hd_p/k_B, particle Nusselt number;
t,	contact time;
T,	emulsion phase temperature;
V,	volume;
$V(x)$,	volume of the particle segment—equation (23);
W,	dimensionless temperature defined by equation (7);
$W(Z)$,	dimensionless emulsion phase temperature at point Z;
x,	distance from the surface;
Z,	x/d_p, dimensionless distance from the surface;
ΔZ,	step size.

Greek symbols

β,	solid concentration;
$\beta(x)$,	solid concentration in emulsion phase at point x;
ε,	voidage;
$\varepsilon(x)$,	voidage of emulsion phase at point x;
κ,	thermal diffusivity;
ρ,	specific density;
$\rho(x)$,	specific density of emulsion phase at point x;
τ,	mean packet residence time.

Subscripts

B,	uninfluenced by the constraining surface;
G,	gas;
i,	instantaneous value at time t;
m,	time-mean value;
P,	particle;
W,	wall, heat-transfer surface.

INTRODUCTION

ALTHOUGH many investigations of heat transfer between fluidized beds and immersed surfaces have been carried out, the designer of fluidized bed heat-transfer systems must still resort to approximate empirical equations in order to estimate the coefficient of heat transfer. Measurements of heat transfer between fluidized beds and boundary surfaces have been carried out by many investigators and extensive experimental data are available in the literature.

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A number of mathematical models have been proposed for the prediction of heat-transfer coefficients, the most useful type being based upon transient conduction between the particles and the surface.

None of the models give agreement with experimental data, even under closely controlled residence time conditions unless some empiricism is invoked, although when coupled with empirical equations, most models are capable of fitting data gathered under wide ranges of conditions. The model proposed in this paper has the advantage over the previous models that it can fit most of the available controlled residence time data without any *a posteriori* empirical constant being used.

Nowadays, when considering the mathematical models, two basic philosophies can be discerned. Both methods use a two-phase description of the system but for each approach the characteristics and the function of the phases are different.

The first approach is to regard the bed as a fundamental system consisting of a continuous phase (provided by the fluidizing medium) and a discrete phase which is provided by the solid particles and sets out to solve the transient heat-transfer equations for single particles during their residence at the surface. The second approach uses the analogy between a fluidized bed and a liquid, so considers the emulsion phase to be the continuous phase and the gas bubbles to be the discrete phase. The transient conduction equations are solved for a packet of emulsion swept up to the wall by the bubbles, a stirrer, or by flowing the particles over the surface. Each of these approaches has some inherent advantages over the other.

BASIC MODELS, THEIR LIMITATIONS, REFINEMENTS AND EXTENSION

Single particle models

The simplest model based on this approach was developed by Botterill and Williams [1], who assumed that an isolated particle surrounded by gas contacts the heat-transfer surface for a certain time, during which the heat is transferred to it by transient conduction. The solution was necessarily numerical and the experimental data, gathered from closely controlled residence time experiments, deviated greatly from the prediction. This shortcoming was removed by the expedient of introducing a gas film between the particle and the surface. With this adjustment good agreement between predicted and experimental data was possible for short particle residence times. This is the major limitation on the use of this type of model, although, even if the position and residence time of a particle near the surface were known precisely, there are still mathematical limitations in the basic model. The model will be accurate only if the heat from the heat-transfer surface does not penetrate beyond the first layer of particles. The depth of heat penetration is given in reference [2]:

$$L_d \propto (\kappa_B t)^{1/2}$$

$$\therefore \frac{L_d}{d_p} \propto (Fo)^{1/2}$$

where κ_B is the thermal diffusivity of the emulsion. Thus, the single particle approach can be expected to be accurate at low Fourier numbers only. This limitation has been long recognized, and Botterill and Butt [3] and Gabor [4] have solved the heat diffusion equations for additional layers of particles. However, the mathematical modelling of single layers of particles has limitations and it is doubtful if the extension of the domain to further layers can be justified physically.

Emulsion phase models

The models based upon the liquid analogy stem from the work of Mickley and Fairbanks [5]. They consider a packet of constant voidage emulsion phase to be swept into contact with the heat-transfer surface for a period of time. During this time, known as the packet residence time, the heat is transferred by non-steady conduction at the surface until the packet is replaced by a fresh packet. Even assuming that all of the thermophysical properties of the packet can be determined accurately, the model still has limitations. Thus, at the heat transfer surface the packet properties must differ from those in the bed due to the voidage variation. For the surface layer effect to be negligible, the heat penetration depth must be much greater than the layer of altered thermophysical properties, which is usually of order one particle diameter. This model should be accurate for large values of the Fourier number. At small values, however, the model fails.

Models based on this penetration theory approach have been considerably refined to extend their validity to low Fourier numbers. The methods used are to introduce a time-independent contact resistance at the bed–surface interface to account for the increased voidage in the vicinity of the surface [6], or to consider the packet to have a limited depth (so setting a boundary constraint) [7].

The concept of a time independent contact resistance appears to be a good first approximation to the problem as it enables good curve fitting of the available data [8]. However, the physical reasoning behind it is somewhat tenuous and its introduction the result of mathematical expedience [4]. The evaluation of this contact resistance is based upon empirical considerations which enable a good data-fit to be obtained.

In conclusion, each of the basic approaches has limitations; the former approach being reasonable at low values of the Fourier number, the latter at large values. Extensions of these asymptotic solutions rely upon empirical considerations which somewhat reduce their theoretical validities. The model developed here uses the penetration theory on the emulsion phase with a modification to allow for variable properties in the region of the surface by introducing the concept of a property boundary layer.

PROPOSED MODEL FOR SURFACE TO BED HEAT TRANSFER

Assumptions in the theoretical treatment

The assumptions are similar to those introduced by Mickley and Fairbanks [5]:

(i) The dense phase has a constant voidage and is isothermal in the bulk of the bed.

(ii) Packets of dense phase are transferred to the heat-transfer surface either by bubble induced circulation, stirring or flowing of the solids. The heat-transfer mechanism is one of transient conduction during the time of packet residence at the surface.

(iii) The only constraint on the position of particles at the surface is provided by the surface itself [9], which influences the local packing and hence alters the local thermophysical properties.

(iv) The variation of voidage is confined to the plane normal to the wall.

Formulation of the model

Considering the heat transfer to be in the direction normal to the wall in the emulsion phase, the semi-infinite layer approximation can be applied and the Fourier equation takes the form:

$$\rho(x)c_p(x)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial T}{\partial x}\right). \tag{1}$$

With boundary conditions for $t \geqslant 0, x \geqslant 0$:

$$t \geqslant 0, x = 0 \quad -k(x)\frac{\partial T}{\partial x} = f_s \tag{2}$$

$$t = 0, x \geqslant 0 \quad T = 0. \tag{3}$$

These equations can be written in dimensionless form

$$\frac{\partial W}{\partial Fo} = H(Z)\frac{\partial^2 W}{\partial Z^2} + L(Z)\frac{\partial W}{\partial Z} \tag{4}$$

subject to

$$Fo \geqslant 0, Z = 0 \quad \frac{\partial W}{\partial Z} = -1/b(Z) \tag{5}$$

$$Fo = 0, Z \geqslant 0 \quad W = 0 \tag{6}$$

by using the following transformations:

$$W = Tk_B/f_s d_p \tag{7}$$

$$Z = x/d_p \tag{8}$$

$$Fo = \kappa_B t/d_p^2 \tag{9}$$

$$g(Z) = \rho(Z)c_p(Z)/(\rho c_p)_B \tag{10}$$

$$b(Z) = k(Z)/k_B \tag{11}$$

$$H(Z) = b(Z)/g(Z) \tag{12}$$

$$L(Z) = \frac{db(Z)}{dZ}\bigg/ g(Z). \tag{13}$$

In order to solve equations (4)–(6) the functions defined in (10)–(13) must be evaluated. These evaluations require a knowledge of the voidage variation in the vicinity of the surface.

Voidage variation in the vicinity of a constraining surface

The variation of local voidage of packed beds in the vicinity of a constraining wall has been investigated by several workers [9–11]. Using spherical particles the usual observation is that the voidage variations with distance from the constraining surface take the form of a damped oscillation curve, having a minimum voidage at about one particle radius from the surface. In the case of fluidized beds the oscillations of the voidage appear to be damped much more rapidly [11] and the voidage minimum is shifted further from the constraining surface. This situation is further exaggerated for non-spherical particles, the voidage remaining practically constant after one particle diameter and having a minimum at $0{\cdot}75\,d_p$ from the surface.

Particles at the same distance from the constraining surface will be influenced by it in the same way. Hence, the mean voidage of any plane parallel to the constraining surface is a function of the distance of the plane from the constraining surface. The area voidage of these planes is

$$\varepsilon(x) = 1 - \beta(x) \tag{14}$$

where $\beta(x)$, the solid concentration is given by

$$\beta(x) = \frac{A(x)}{A_B}. \tag{15}$$

$A(x)$ is the solid cross-sectional area at a reference plane which is situated at a distance x from the constraining surface. In order to calculate the particle cross-sectional area at any distance the following assumption is made.

The particles in the bed can be approximated by uniform spherical particles having the mean equivalent diameter of the actual particles without affecting the voidage distribution within the packet.

It is now postulated that for a bed of uniform spherical particles, the solid cross-sectional area $A(x)$ at a reference plane distance x from the surface is proportional to the cross-sectional area of a cylinder whose volume and height are identical to those of the segment of a spherical particle which can be found between the reference plane and the surface while touching the surface (Fig. 4).

When the reference plane is situated beyond one particle diameter from the surface, the fraction of a spherical particle in contact with the surface which can be found between it and the surface is constant, and equal to the entire particle. Thus, it is implied that the asymptotic material cross-section is attained at a distance of one particle diameter from the wall. The voidage variation can then be calculated

$$x \leqslant d_p \quad A(x) \propto V(x)/x \tag{16}$$

$$x > d_p \quad A(x) \propto V_p/d_p \tag{17}$$

where $V(x)$ is the volume of that segment of a spherical particle between the surface and the reference plane and V_p is the particle volume.

Equations (16) and (17) can be rewritten

$$x \leqslant d_p \quad \beta(x) = CV(x)/x \tag{18}$$

$$x > d_p \quad \beta(x) = CV_p/d_p \tag{19}$$

where C is a constant of proportionality. The r.h.s. of equation (19) is a constant uninfluenced by the constraining surface equal to the material concentration within the emulsion phase, β_B. Hence

$$CV_p/d_p = 1-\varepsilon_B = \beta_B. \tag{20}$$

Then from equations (18) and (19)

$$x \leqslant d_p \quad \beta(x) = (1-\varepsilon_B)\frac{d_p}{x}\frac{V(x)}{V_p} \tag{21}$$

$$x > d_p \quad \beta(x) = 1-\varepsilon_B \tag{22}$$

where $V(x)$, the volume of the segment between the surface and the plane, is given by:

$$V(x) = \tfrac{1}{3}\pi x^2(\tfrac{3}{2}d_p - x). \tag{23}$$

Equations (21) and (22) can be written in dimensionless form:

$$Z \leqslant 1 \quad \varepsilon(Z) = 1-3(1-\varepsilon_B)(Z-\tfrac{2}{3}Z^2) \tag{24}$$

$$Z > 1 \quad \varepsilon(Z) = \varepsilon_B. \tag{25}$$

The predicted variation of the voidage for various emulsion voidages, ε_B, is plotted in Fig. 1. These predictions agree well with the experimental data of [9] and [11].

Equations (24) and (25) can be used to calculate the variables $H(Z)$ and $L(Z)$.

The functions H(Z) *and* L(Z)

(i) When the fluidizing medium is a gas, its heat capacity can be neglected relative to that of the solids so that

$$\rho(Z)c_p(Z) = (\rho c_p)_p\beta(Z) \tag{26}$$

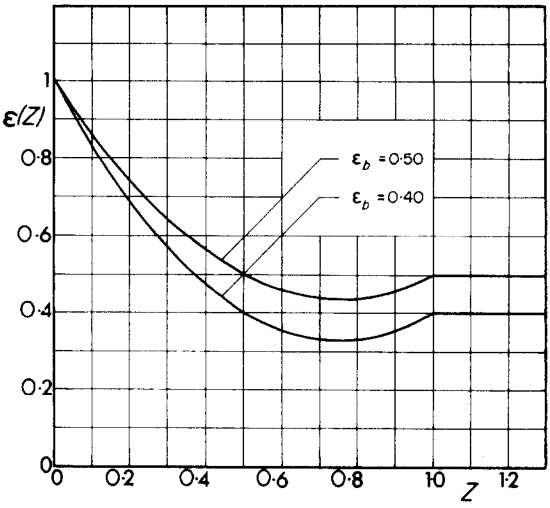

FIG. 1. Variation of the dense phase voidage in the vicinity of a flat surface.

and

$$(\rho c_p)_B = (\rho c_p)_p\beta_B \tag{27}$$

and hence

$$g(Z) = \beta(Z)/\beta_B. \tag{28}$$

(ii) The effective conductivity of a packet can be calculated from one of the models available in the literature. In this work the method proposed by Kunii and Smith [12] is used and it is assumed that its validity can be extended into regions of high voidage [13]. The local effective thermal conductivity of a packet can be expressed in the form:

$$k(Z) = fn[k_p, k_G, \varepsilon(Z)]. \tag{29}$$

Using equations (29) and (11), $b(Z)$ can be readily calculated. However, $k(Z)$ can be more easily evaluated graphically than using its explicit form [12]. Figures 2 and 3 show the behaviour of function $b(Z)$ for various ratios of gas and particles thermoconductivities and two values of the mean packet voidage, ε_B.

(iii) The function $H(Z)$ is then calculated from equation (12). In order to calculate the function $L(Z)$, $b(Z)$ must be differentiated, in this work by numerical differentiation. $L(Z)$ is then calculated from equation (13).

THEORETICAL HEAT-TRANSFER COEFFICIENTS

Numerical calculations

Equations (4)–(6) were solved numerically using the voidage distribution given by equations (24) and (25). The calculated thermophysical properties vary very rapidly close to the heat-transfer surface which type

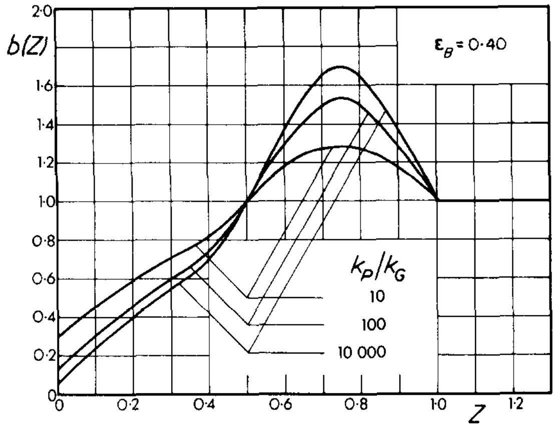

FIG. 2. Variation of the dimensionless conductivity, $b(Z)$, with Z for dense phase voidage $\varepsilon_B = 0{\cdot}40$.

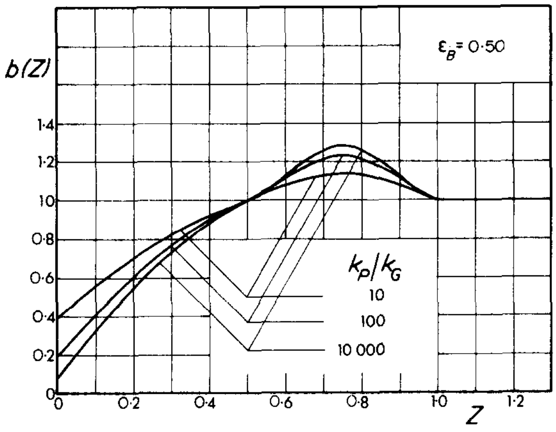

FIG. 3. Variation of the dimensionless conductivity, $b(Z)$, with Z for dense phase voidage $\varepsilon_B = 0{\cdot}50$.

of variation lends itself most readily to constant flux boundary conditions. The physical situation is somewhere between constant wall temperature and constant wall flux. However, the solutions obtained by either approach are almost the same [2]. In order to solve the equations accurately, very small space (and hence time) increments must be used because of these large variations close to the surface. The size of increments needed makes computer time requirements extremely large.

In order to overcome this complication while not sacrificing much accuracy, it was assumed that the first space increment from the surface remained at steady state so that the situation is as shown in Fig. 4 and the wall temperature, $W(0)$, can then be calculated as shown in the Appendix, from,

$$W_W = W(0) = W(\Delta Z) + \frac{\Delta Z}{b_{\text{eq}}} \tag{30}$$

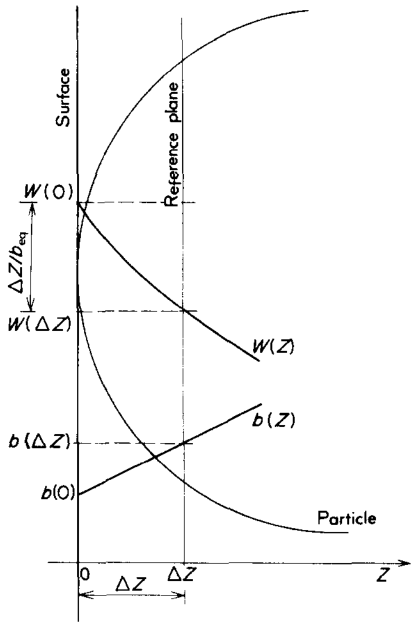

FIG. 4. Considerations for deriving b_{eq}, used in the numerical calculation.

where,

$$b_{\text{eq}} = \frac{b(\Delta Z) - b(0)}{\ln \dfrac{b(\Delta Z)}{b(0)}}. \tag{31}$$

The temperature field in the packet was calculated by a standard numerical technique [2] until that distance from the surface at which the dimensionless temperature, W, became smaller than 10^{-10}; at this point the calculation was terminated and restarted for the next time increment. The space incremental size used was ΔZ equals 0·05. The solution thus obtained was checked for accuracy by halving and quartering the step-size. This demonstrated both the adequacy of the step-size and that the simplification [equations (30) and (31)] was numerically reasonable, leading to a great saving in computer time while incurring negligible loss of accuracy.

The instantaneous Nusselt number

An instantaneous Nusselt number can be defined as

$$Nu_i = \frac{h_i d_p}{k_B} \tag{32}$$

where h_i is the instantaneous heat-transfer coefficient which can be expressed,

$$h_i = \frac{f_s}{(T_W)_i} \tag{33}$$

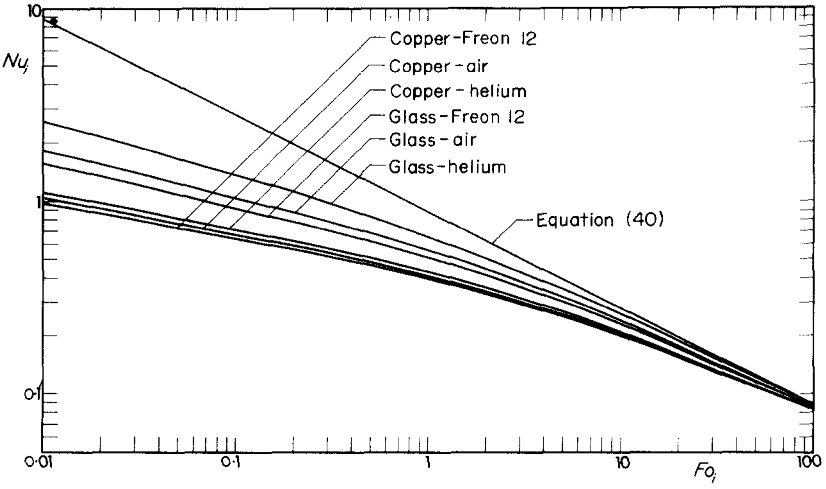

FIG. 5. Instantaneous Nusselt number, Nu_i, as a function of the instantaneous Fourier number, Fo. Voidage of the dense phase is taken as $\varepsilon_B = 0{\cdot}41$.

so that,

$$Nu_i = \frac{d_p f_s}{k_B (T_W)_i}. \tag{34}$$

Thus, from equation (7),

$$Nu_i = \frac{1}{(W_W)_i}. \tag{35}$$

Figure 5 shows the variation of the instantaneous Nusselt number with the Fourier number.

Time-mean Nusselt number

The time-mean heat-transfer coefficient (and hence time-mean Nusselt number) can be calculated if the mean residence time and the residence time distribution functions are known. Thus,

$$h_m = \frac{f_s}{(T_W)_m}. \tag{36}$$

In order to evaluate $(T_W)_m$ a residence time distribution function must be used. The simplest form for fluidized beds appears to be the uniform residence time distribution function [6, 7] although many other forms can be used [14, 15]. The effect of the distribution function selected is not numerically large [7, 19], so that

$$(T_W)_m = \frac{1}{\tau}\int_0^\tau (T_W)_i \,\mathrm{d}t \tag{37}$$

where τ is the periodic time of renewal of packets on the heat-transfer surface. The mean Nusselt number is

$$Nu_m = \frac{1}{(W_W)_m} \tag{38}$$

where

$$(W_W)_m = \frac{1}{Fo_m}\int_0^{Fo_m} (W_W)_i \,\mathrm{d}Fo. \tag{39}$$

The theoretical predictions can be compared with experimental results obtained under closely controlled conditions such as in flowing or in stirred beds. Figure 6 shows the variation of the time-mean Nusselt number with the time-mean Fourier number.

COMPARISON WITH EXPERIMENTAL DATA

The discrepancies amongst the available data, measured in freely bubbling beds under apparently identical conditions of fluidization, can be of several orders of magnitude [16], reflecting the many parameters which can influence fluidized bed heat-transfer rates. Hence, in order to test the various theories or develop new models, simplified systems in which the bed behaviour is closely controlled are used. The experimental systems can be classified as follows:

(i) *The low thermal capacity probe*

In this technique a small, low heat capacity heating foil is used to measure both heat-transfer coefficients and packet residence times in freely bubbling beds [17]. While this technique is very useful, the results obtained are subject to errors because the surface temperature and flux vary during any test and further, the packet voidage at the surface is distorted by the passing bubbles.

(ii) *Stirred beds*

The heat-transfer probe is moved through the bed on a stirrer [18] or the bed is stirred past a heat-transfer probe [1] in order to effect residence time control. However, this technique involves two questionable assumptions. Firstly, particle replacement over the entire surface is complete and secondly, the bed

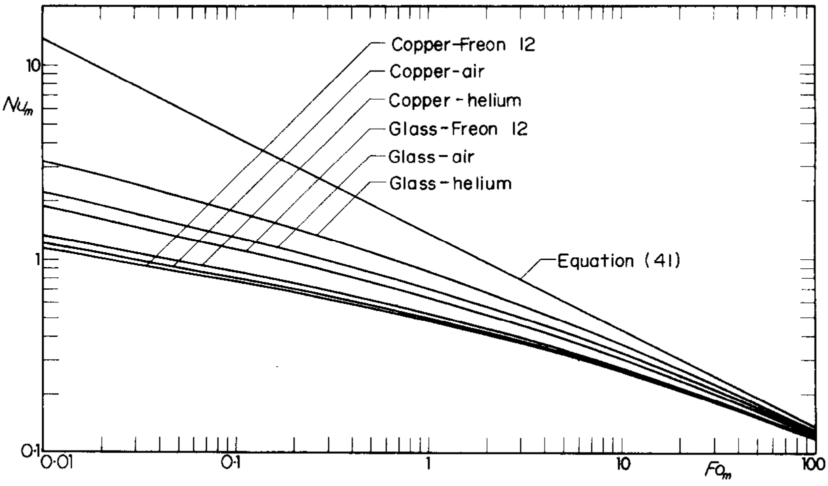

FIG. 6. Time–mean Nusselt number, Nu_m, as a function of the time–mean Fourier number, Fo_m. Voidage of the dense phase is taken as $\varepsilon_B = 0{\cdot}41$.

structure is unaffected by the stirring. Each of these assumptions has been shown to be of limited validity [4].

(iii) *Flowing packed beds*

According to the two-phase theory of fluidization the dense phase of a fluidized bed is considered to be under approximately incipiently fluidized conditions. The incipiently fluidized bed has many of the characteristics of a packed bed [20] so that the dense phase can be simulated for many purposes by a packed bed. Many data have been gathered for packed beds flowing over heat-transfer surfaces. The main experimental limitation of this technique is that particle rotation has been observed under some conditions [21].

(iv) *Transient response*

In this technique a heat-transfer probe of known heat capacity and initial temperature is submerged into an incipiently fluidized or packed bed [22]. The variation of heat-transfer coefficient with time, obtained from the temperature response of the probe, is analogous to the response of a packet of particles at a heat-transfer surface in a fluidized bed over the same time interval. This technique suffers from the same disadvantage as the first method in that both flux and temperature vary at the surface. However, the packing at the surface is not in this case disturbed by passing bubbles.

Experimental data, obtained under all of the above conditions are available in the literature. In comparing these data with the prediction of this work it was sometimes necessary to use assumed values for the dense phase voidage to calculate the required packet properties. The voidage then used was 0·41 and the conductivity was calculated as outlined earlier. The properties of the particles and gases used in this comparison are set out in Table 1.

Table 1. Physical properties at 24°C

	k (W/m K)	c_p (J/kg K)	ρ (kg/m^3)
Air	0·026	1008	1·19
Helium	0·148	5200	0·165
Freon 12	0·0097	650	5·14
CO_2	0·016	850	1·79
Glass	0·850	765	2700
Copper	380	385	8930
Steel	45	480	7800
Silica sand	1·87	860	2600
Slag [18]	0·59	752	2720

Instantaneous Nusselt numbers

The prediction of the present theory is compared with the results of Antonishin *et al.* [22] in Fig. 7. The asymptotic Mickley–Fairbanks solution, assuming a constant flux boundary condition, and given by Carslaw and Jaeger [2] as

$$Nu_i = \frac{\sqrt{\pi}}{2} Fo^{-1/2} \tag{40}$$

is also included.

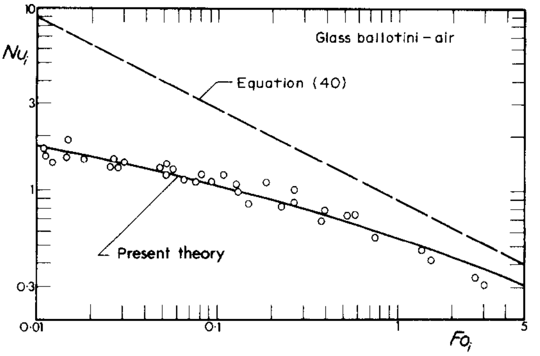

FIG. 7. Comparison of experimental data with present theory for instantaneous heat transfer in the system glass ballotini–air. (Experimental data of Antonishin *et al.* [22].)

Time-mean Nusselt numbers

The experimental results of a number of workers are compared with the present theory in Figs. 8–11. Again, the Mickley–Fairbanks solutions calculated from equations (36), (37) and (40) as

$$Nu_m = \frac{3\sqrt{\pi}}{4} Fo_m^{-1/2} \tag{41}$$

are included.

DISCUSSION

As the present model involves the use of the experimental results of several workers, for example in calculating voidage variations near surfaces or packet properties, it is very satisfying to see the very good agreement between the present theory and the controlled residence time data available in the literature. It is significant that the best agreement between theory and experiment was obtained using the results from experiments conducted under very carefully controlled conditions with materials of well known properties in the emulsion phase. This suggests that where the disagreement is most pronounced, the true experimental conditions may not have been measured. These errors may be due to:

(i) The stirrer not being 100 per cent efficient.
(ii) In flowing packed bed experiments some particle rotation could occur.
(iii) The voidage was not reported for many of the experiments and the value of 0·41 may not be correct.
(iv) The calculated thermal conductivities of packets is probably the biggest single source of error. This error is indicated when the results of a particular study fall systematically either above or below the theoretical line.

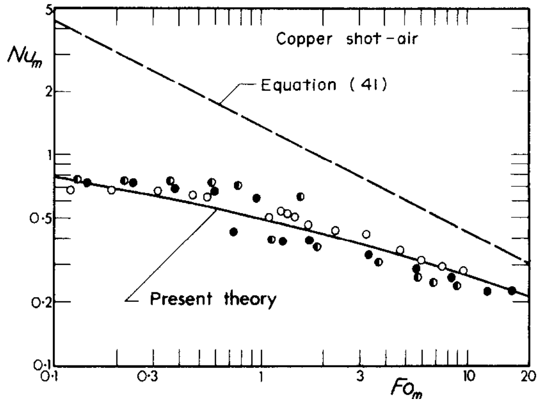

FIG. 8. Comparison of experimental data with present theory for time–mean heat transfer in the system copper shot–air. (Experimental data of Butt [26], ○ ; Hampshire [24], ◐ ; and Desai [27], ●.)

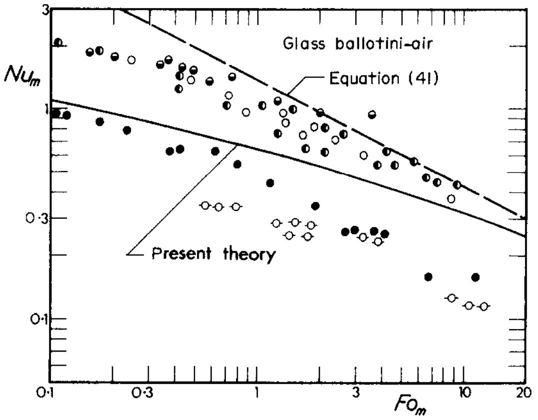

FIG. 9. Comparison of experimental data with present theory for time–mean heat transfer in the system glass ballotini–air. (Experimental data of Harakas and Beatty [28], ○ ; Williams and Smith [29], -○-; Hampshire [24], ⊖; Butt [26], ◐; and Cain [25], ●.)

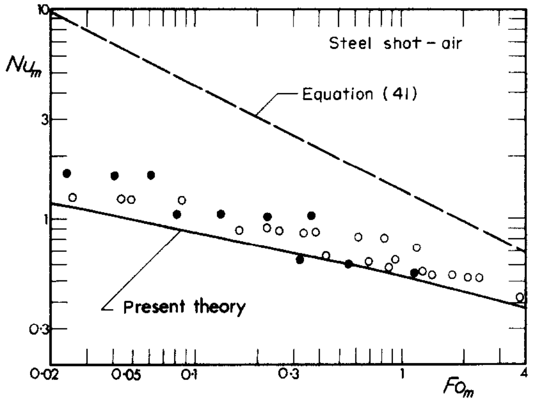

FIG. 10. Comparison of experimental data with present theory for time–mean heat transfer in the system steel shot–air. (Experimental data of Butt [26], ○; and Hampshire [24], ●.)

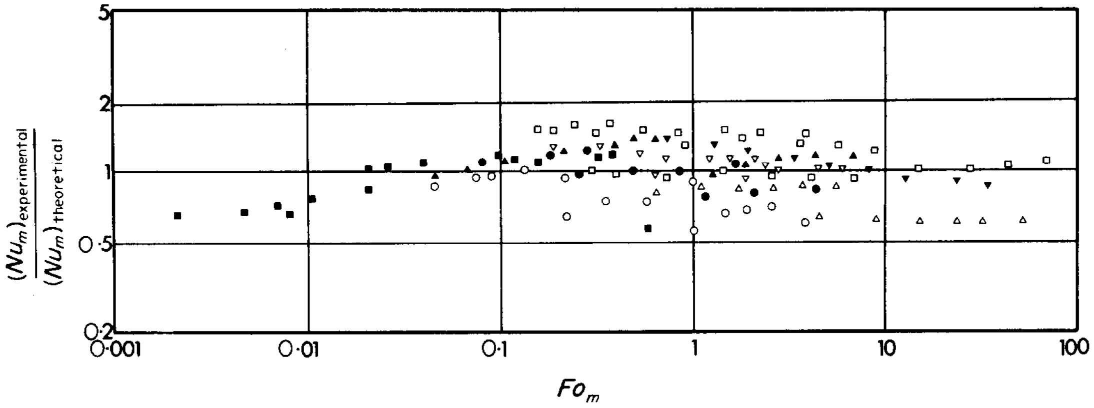

FIG. 11. Comparison of experimental data with present theory for time-mean heat transfer in the following systems: △, copper shot–helium [24]; ▲, copper shot–carbon dioxide [27]; ▽, glass ballotini–helium [24]; ▼, glass ballotini–helium [28]; ○, glass ballotini–carbon dioxide [25]; ●, steel shot–helium [24]; □, quartz sand–air [30]; ■, slag spheres–air [18].

(v) In the case of fluidized metals a further complication is that an oxidized microlayer on the surface of the particle would considerably effect the conductivity of a packet.

These effects could be eliminated by calculating the effective conductivity of the emulsion phase from heat-transfer results with long packet residence times, using Mickley and Fairbanks asymptotic solution. However, the agreement obtained without using this method seems to make such a refinement unnecessary.

None of the previous models of heat transfer in fluidized beds have been capable of producing acceptable agreement with the experimental data over the full range of data without the introduction of some form of semi-empirical approximation. While these approximations, such as a gas-film, a region of reduced voidage or a finite penetration depth, have obvious physical significance, they are all *ad hoc* expedients rather than solutions to the real problem of behaviour close to a surface. The model developed here uses no assumed film properties and is analogous to the variable wall properties approach long used in single phase heat transfer, such as the Sieder–Tate correction for viscosity variations [23].

Voidage variations are confined to a distance of one particle diameter from the surface and can be described accurately from simple geometrical considerations.

Figures 7–11 show that the model developed here gives a good agreement with experimental data where conduction is the predominant mode of heat transfer. In principal, this model could be extended to more complicated mixed-mode heat-transfer situations, for example to high pressure systems where gas convective transfer is significant [21].

The voidage variation in the vicinity of a surface could probably be described by a more accurate expression than that developed here and the numerical approximation improved. However, it is doubtful whether the increase in precision of the theoretical solution would be justified in terms of the experimental limitations.

The main disadvantage of all of these theoretical models is that they cannot be readily applied to freely bubbling beds where the actual dense phase motion cannot yet be described adequately in any quantitative manner.

CONCLUSIONS

1. The model derived here for heat transfer between a surface and a fluidized bed agrees well with all controlled residence time data available in the literature.
2. The packet theory of heat transfer is modified to take account of the presence of the surface and its effect on the local voidage by introducing a physically justified property boundary layer.
3. The voidage variation near a surface can be described from simple geometrical considerations, and the equation derived agrees well with the available data.
4. The model proposed, using the derived property boundary layer, requires no physically unjustified concepts in order to produce agreement with experimental data.

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APPENDIX

Constant flux conditions at the surface of the heat-transfer probe are given by equation (5) as

$$\frac{\partial W}{\partial Z} = -1/b(0). \tag{A.1}$$

Equation (A.1) is valid only at the surface of the heat-transfer probe. If a finite difference numerical method is used with a finite material slab, equation (A.1) is not applicable and must be modified to

$$\frac{\partial W}{\partial Z} = -1/b_{\mathrm{eq}} \tag{A.2}$$

where b_{eq} is the equivalent thermal conductivity (dimensionless) of the first slab (Fig. 4).

The temperature at the wall can be calculated from the temperature at point ΔZ as [2]

$$W(0) = \frac{\Delta Z}{b_{\mathrm{eq}}} + W(\Delta Z). \tag{A.3}$$

Equation (A.3) is based on the assumption that the first slab of material remains at steady state throughout the heating process. The same assumption can be applied on a material slab of variable conductivity. This is done by solving

$$\frac{\mathrm{d}}{\mathrm{d}Z}\left[b(Z)\frac{\mathrm{d}W}{\mathrm{d}Z}\right] = 0 \tag{A.4}$$

subject to

$$Z = 0 \quad -b(0)\frac{\mathrm{d}W}{\mathrm{d}Z} = 1 \tag{A.5}$$

$$Z = \Delta Z \quad W = W(\Delta Z). \tag{A.6}$$

It is further assumed that, because the slab thickness is small compared with the particle diameter, the variation of dimensionless thermal conductivity can be approximated by a linear function satisfying the following conditions

$$Z = 0 \quad b(Z) = b(0) \tag{A.7}$$

$$Z = \Delta Z \quad b(Z) = b(\Delta Z). \tag{A.8}$$

The function $b(Z)$ can then be approximated in the first slab by

$$b(Z) = b(0) + \frac{b(\Delta Z) - b(0)}{\Delta Z} Z. \tag{A.9}$$

The solution of equation (A.4) with $b(Z)$ given by equation (A.9) is

$$W(Z) = W(\Delta Z) + \frac{\Delta Z}{b(\Delta Z) - b(0)} \ln \frac{\Delta Z + \dfrac{b(0)\Delta Z}{b(\Delta Z) - b(0)}}{Z + \dfrac{b(0)\Delta Z}{b(\Delta Z) - b(0)}} \tag{A.10}$$

from which the wall temperature $W(0)$ can be calculated as

$$W(0) = W(\Delta Z) + \frac{\Delta Z}{b(\Delta Z) - b(0)} \cdot \ln \frac{b(\Delta Z)}{b(0)}. \tag{A.11}$$

Finally, comparing equations (A.3) and (A.11) the expression for the equivalent conductivity of the first slab is given as

$$b_{\text{eq}} = \frac{b(\Delta Z) - b(0)}{\ln \dfrac{b(\Delta Z)}{b(0)}}. \tag{A.12}$$

UN MODELE DE TRANSFERT DE CHALEUR DANS LES LITS FLUIDISES GAZEUX

Résumé—On discute du mécanisme du transfert thermique aux surfaces dans des lits fluidisés et on développe une variante de la théorie de pénétration dans laquelle la théorie du transfert de chaleur est modifiée pour tenir compte et décrire quantitativement des variations de propriétés dans la région proche de la surface.

Les variations de propriétés sont décrites en fonction des variations du taux de vide, au voisinage d'une surface contraignante, qui sont modélisées à partir de simples considérations géométriques.

Le modèle du processus de transfert thermique établi ici, montre un bon accord avec les données expérimentales, sans recours à aucune approximation semi-empirique comme rencontrée généralement dans les modèles antérieurs.

EIN MODELL FÜR DEN WÄRMEÜBERGANG IN GASFLIESSBETTEN

Zusammenfassung—Der Wärmeübergangsmechanismus an der Oberfläche von Fließbetten wird diskutiert und eine Variante der Penetrationstheorie entwickelt, in der die Festbettheorie für den Wärmeübergang modifiziert wird, um die Beschreibung von Eigenschaftsvariationen der Packung in der Region der Oberfläche zu ermöglichen und quantitativ zu erfassen. Die Eigenschaftsvariationen werden in Termen der Lückenvariationen in der Nähe einer erzwungenen Oberfläche beschrieben, die aus einfachen geometrischen Betrachtungen gewonnen werden.

Das hier entwickelte Wärmeübergangsmodell zeigt gute Übereinstimmung mit experimentellen Daten ohne Bezugnahme auf halb-empirische Näherungen, die Merkmal früherer Modelle sind.

МОДЕЛЬ ТЕПЛООБМЕНА В ПСЕВДООЖИЖЕННЫХ ГАЗАМИ СЛОЯХ

Аннотация — Обсуждается механизм теплообмена на поверхности, омываемой псевдоожиженным слоем, и предлагается вариант теории «проникновения», в которой пакетная теория теплообмена модифицирована для учета и количественного описания изменения свойств пакета вблизи поверхности.

Изменения свойств описываются через изменение порозности слоя вблизи омываемой им поверхности, причем использованы простые геометрические соображения.

Предлагаемая модель теплопереноса хорошо согласуется с экспериментальными данными, не требуя никаких полуэмпирических приближений характерных для прежних моделей.